4.5.5 Wear Model

The model of Finnie for erosive wear [20] relates the rate of wear to the rate of kinetic engery of particle impact on a surface. It reads

$$E = k\, v_p^2\, f(\gamma). \tag{4.16}$$

where E is a dimensionless mass, v_p is the magnitude of the particle impact velocity, k is a constant and $f(\gamma)$ is a dimenless function of the impact angle between the approaching track and the surface:

$$f(\gamma) = \frac{1}{3} cos^2(\gamma) \quad if\, tan(\gamma) > \frac{1}{3} \tag{4.17}$$

$$f(\gamma) = sin(2\gamma) - 3\, sin^2(\gamma) \quad if\, tan(\gamma) < \frac{1}{3} \tag{4.18}$$

This model formulation is for use in the frame of the hard sphere model. When used in conjunction with a Lagrangian particle tracks representing a mass flow rate of $\dot{m}$, the eroded mass EM in kg during one time-step Δt becomes

$$EM_{HS} = E\, \dot{m} \Delta t. \tag{4.19}$$

We now adapt the model for use in the frame of the soft sphere model. Since the soft sphere model resolves a particle-surface contact with multiple time-steps instead of one step, we propose the following relation for the eroded mass caused by one particle during the particle - surface contact:

$$EM_{SS} = \int_0^{tc} e\, dt\, m_p. \tag{4.20}$$

tc is the contact time, m_p is the particle mass, and

$$e = \frac{dE}{dt} = \frac{\partial E}{\partial v_p} \frac{\partial v_p}{\partial t} + \frac{\partial E}{\partial \gamma} \frac{\partial \gamma}{\partial t}. \tag{4.21}$$

is a is the time derivative of the dimensionless mass. Since $\frac{\partial v_p}{\partial t}$ will be the clearly dominant part of the time derivative and tracking $\frac{\partial \gamma}{\partial t}$ would be computationally tedious, we assume $\frac{\partial \gamma}{\partial t} \approx 0$. Hence we get

$$e \approx \frac{\partial E}{\partial v_p} \frac{\partial v_p}{\partial t} = \frac{2\, E}{v_p} \frac{f_c}{m_p}. \tag{4.22}$$

with f_c being the particle-surface contact force. Furthermore we postulate $e = 0$ for $\underline{v_p}\,\underline{c} < 0$, where $\underline{c}$ is the vector pointing from the particle's center to the contact point. This is because re assume the wear to be caused duing the impact phase of the contact, not during the repulsion phase. Finally we get

$$EM_{SS} = 2\,k \int_0^{tc} hs(\underline{v_p}\,\underline{c})\, v_p\, f(\gamma)\, f_c\, dt. \tag{4.23}$$

with hs being the Heaviside function.